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A DETERMINISTIC INVENTORY MODEL FOR DETERIORATING ITEMS WITH ON-HAND INVENTORY DEPENDENT, QUADRATIC DEMAND RATE

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ABSTRACT

An infinite time horizon model for deterministic perishable items that follows quadratic demand rate, constant deterioration and without shortages is considered. An optimal production strategy is derived with maximization of profit. The result is illustrated with numerical example**.**

KEYWORDS: On-Hand Inventory, Replenishment Cost, Shortages, Variable Demand

1. INTRODUCTION

In present trends businessmen have shown very much alertness of the need for precision in the field of inventory control of deteriorating items. In general, deterioration is defined as damage, obsolescence, evaporation, spoilage, decay, pilferage etc. that result in decrease of usefulness of the original one. In general a product may be understood to have a lifetime which ends when utility reaches zero. The decrease or loss of utility due to decay is generally a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as alcohol, gasoline, radioactive chemical, blood, fish, strawberry, medicine and food grains (i.e., paddy, wheat, potato, onion etc.) deteriorate surprisingly overtime. Whitin [5] considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Ghare and Scharder [7] developed an EOQ model with an exponential decay and a deterministic demand. Thereafter, Covert and Philip [3] and Philip [10] extended EOQ (Economic Order Quantity) models for deterioration which follows Weibull distribution. Wee [9] developed EOQ models to allow deterioration and an exponential demand pattern. In last two decades the economic situation of most countries have changed to an extent due to sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of time value of money. Data and Pal [1], Bose et al. [2] have developed the EOQ model incorporating the effects of time value of money, a linear time dependent demand rate. Further Sana [8] considered the money value and inflation in a new level. In the present paper demand rate is considered as constant to a fixed time, after then it varies linearly with time. Sahu et al [6] and Samal et al [4] have established models where the demand rate is dependent on the on-hand inventory. In what follows in the present paper we consider a model by taking variable type of demand which behaves differently in the given time horizon such as the demand rate is constant for a certain fixed time and after that period it varies linearly with time.

2. FUNDAMENTAL ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

- Lead time is zero.
- Replenishment rate is infinite but size is finite.
- Demand rate is variable with respect to time.
- Shortages are not allowed.
- Single inventory will be used.
- Time horizon is finite.
- There is no repair of deteriorated items will occur during the cycle.

Following notations are made for the given model:

$R(t)$ = Demand.

R_0 = Initial demand rate.

$I(t)$ = On hand inventory at time t.

I_0 = Inventory at time $t = 0$.

Q = On-hand inventory.

T = Duration of a cycle.

θ = The constant deterioration rate where $0 \leq \theta \leq 1$.

s = Selling price per unit.

c = Unit cost of the item.

h = Holding cost per unit item per unit time.

r = Replenishment cost per replenishment which is a constant.

a = Rate of change of demand with respect to t.

3. FORMULATION

In this model we consider the rate of demand $R(t)$ to be a constant up to a certain time $t = t_1$ and after which it varies quadratic with time. If $I(t)$ be the on hand inventory at time $\tau \geq 0$, then at time $t + \Delta t$, the on-hand inventory will be

$$I(t + \Delta t) = I(t) - R(t)\cdot\Delta t - \theta\cdot I(t)\cdot\Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

$$\frac{dI}{dt} = -R(t) - \theta \cdot I(t) \qquad (1)$$

We define $R(t)$as

$$R(t) = R_0 + a(t - t_1)^2 H(t - t_1) \quad (2)$$

Where $H(t - t_1) = 1$ for $t \geq t_1$ and 0 for $t \leq t_1$

From equation (2) we have

$$\frac{dI}{dt} = -R_0(t) - \theta \cdot I(t) \text{ for } 0 \leq t \leq t_1 \quad (3)$$

$$\frac{dI}{dt} = -R_0 - a(t - t_1)^2 - \theta \cdot I(t) \text{ for } t_1 \leq t \leq T \quad (4)$$

We observe that the equation (3) is linear equation solving it we have

$I = -\dfrac{R_0}{\theta} + Ce^{-\theta t}$, Where C is the constant of integration

Using the initial condition $I = I_0$ at $t = 0$ we have

$$I = -\frac{R_0}{\theta} + e^{-\theta t}\left(I_0 + \frac{R_0}{\theta}\right) \text{ for } 0 \leq t \leq t_1 \quad (5)$$

Again using $I = I_1$ at $t = t_1$

$$I_1 = -\frac{R_0}{\theta} + e^{-\theta t_1}\left(I_0 + \frac{R_0}{\theta}\right) \quad (6)$$

Simplifying equation (6) for t_1 we have

$$t_1 = -\frac{1}{\theta} \cdot ln\left(\frac{R_0 + I_1\theta}{R_0 + I_0\theta}\right) \quad (7)$$

Again we observe that the equation (4) is also linear equation solving it we have

$I(t) = -\dfrac{R_0}{\theta} - \dfrac{a}{\theta^3}[(t - t_1)^2\theta^2 - 2(t - t_1)\theta + 2] + C_1 e^{-\theta t}$, Where C_1 is the constant of integration

Using the initial condition $I = 0$ at $t = T$ we have

$$I = \frac{a}{\theta^2}[(T^2 - t^2)\theta - 2(T - t) - 2t_1\theta(T - t)] \quad (8)$$

But using $I = I_1$ at $t = t_1$

$$I_1 = \frac{a}{\theta^2}[(T^2 - {t_1}^2)\theta - 2(T - t_1) - 2t_1\theta(T - t_1)] \quad (9)$$

From equation (9) we have

$$T = \frac{1}{\theta}\left[1 + t_1\theta + \sqrt{1 + \frac{\theta^3}{a} I_1}\right] \quad (10)$$

Using equation (7) in (10) we have

$$T = \frac{1}{\theta}\left[1 - ln\left(\frac{R_0 + I_1\theta}{R_0 + I_0\theta}\right) + \sqrt{1 + \frac{\theta^3}{a} I_1}\right] \quad (11)$$

Now average on hand inventory can be given by

$$Q = \int_0^T I dt = \int_0^{t_1} I dt + \int_{t_1}^T I dt$$

Now putting the corresponding values of I we have

$$Q = \frac{1}{\theta}\left[\left(I_0 + \frac{R_0}{\theta}\right)\left(1 - e^{-\theta t_1}\right) - R_0 t_1\right]$$

$$+\frac{a}{\theta^2}\left[\frac{2T^3\theta}{3} - 2T^2 t_1\theta + 2{t_1}^2\theta - \frac{2{t_1}^3\theta}{3} - (T - t_1)^2\right] \quad (12)$$

To eliminate the values of t_1 and T we may find,

$1 - e^{-\theta t_1} = \frac{(I_0 - I_1)\theta}{R_0 + I_0\theta}$ and $T - t_1 = \frac{\theta\, I_1}{a}$

Therefore $Q = \dfrac{I_0 - I_1 - R_0 t_1}{\theta} + \dfrac{a}{\theta^2}\left[\dfrac{2T^3\theta}{3} + 2T^2 ln\left(\dfrac{R_0 + I_1\theta}{R_0 + I_0\theta}\right)\right]$

$$+\frac{a}{\theta^2}\left[\frac{2}{\theta}\left\{ln\left(\frac{R_0+I_1\theta}{R_0+I_0\theta}\right)\right\}^2 + \frac{2}{3\theta^2}\left\{ln\left(\frac{R_0+I_1\theta}{R_0+I_0\theta}\right)\right\}^3 - \frac{\theta^2 {I_1}^2}{a^2}\right] \quad (13)$$

Now the profit function $\emptyset(I_0)$ is given by

$$\emptyset(I_0) = \frac{1}{T}\{(s - c - r)I_0\} - \frac{h}{T}\left\{\frac{I_0 - I_1}{\theta} + \frac{\theta {I_1}^2}{2a} + \frac{R_0}{\theta^2} ln\left(\frac{R_0+I_1\theta}{R_0+I_0\theta}\right)\right\} \quad (14)$$

The necessary condition for maximum profit is $\frac{d[\emptyset(I_0)]}{dI_0} = 0$ which gives

$$s - c - r = \frac{h}{\theta}\left(1 - \frac{R_0}{R_0 + I_0\theta}\right)$$

and hence $I_0 = \frac{R_0(s-c-r)}{h - \theta(s-c-r)}$ (15)

Using equation (15)

$$\frac{d^2\emptyset(I_0)}{d{I_0}^2} = \frac{-hR_0}{T(R_0 + I_0\theta)^2} < 0 \quad (16)$$

It will give a global maximum for profit function $\emptyset(I_0)$.

4. NUMERICAL EXAMPLE

We have considered the values of parameters in appropriate units as follows:

(i) $a = 40,\ t_1 = 0.1,\ R_0 = 4.38,\ s = 30,\ c = 10,\ h = 0.9,\ r = 12,\ \theta = 0.1$

Using the above formulas we have $T = 20.14$ years and $\emptyset(I_0) = 130.9$ per year

(ii) $a = 20,\ t_1 = 0.1,\ R_0 = 4.38,\ s = 30,\ c = 10,\ h = 0.9,\ r = 12,\ \theta = 0.1$

Using the above formulas we have $T = 20.19$ years and $\emptyset(I_0) = 123.93$ per year

5. CONCLUSIONS

From the studied physical model, it should be noted that the demand rate of various inventories is remained constant up to a time t_1 after which the demand rate varies with time. During the period $[0, t_1]$, the demand rate is maintained at a constant level but after that period the demand rate varies in quadratic way hence amount of inventory decreases continuously with time also the effect of constant deterioration is maintained throughout the cycle. Hence the inventory level decreases due to the combined effect of demand as well as deterioration. The above model can also be studied under backlogging, shortages and backordering.

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